Reducing Systemic Risk in Multiplex Networks Using Evolutionary Optimization

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Spreading of threats in graphs

Cascading failures

- A system of interconnected entities
- Some of these entities initially fail and these failures overload other entities that also fail
- Examples: power grid failures, a wave of bankruptcies

Coupling strength and phase transitions

- The final number of failures depends on the coupling strength *c* (how much the failure of one node affects the other nodes)
- For lower *c* the failures quickly stop spreading, for higher *c* the failures may affect (almost) all the nodes (catastrophic failure)
- An abrupt change in the behaviour (phase transition) may occur at a critical value of the coupling strength

The Burkholz model[3]

- A two-layer network of companies. Layer 0: core units, layer 1: subsidiary units.
- Connections in each layer: Erdős-Rényi graph (here with N_v = 1000 and P_e = 0.003)
- Some nodes initially fail. The failures affect the neighbouring nodes
  - the load depends on the fraction of the neighbours that have failed
  - thresholds (drawn from U[0, 1]) determine how much load each unit can withstand
- Asymmetric coupling between layers
  - c_{01} = 1: core unit failure induces an instant failure of the subsidiary unit
  - c_{10} < 1: subsidiary unit failure exerts a load on the core unit (lowers its threshold by multiplying by 1 - c_{10})

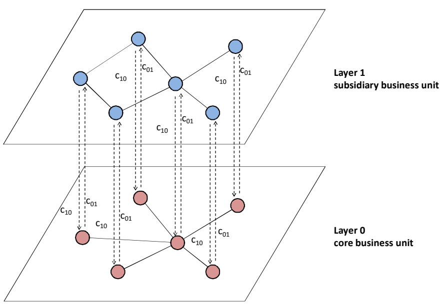

Fig. 1. A two-layer multiplex network representing companies composed of a core-business unit (in layer 0) and a subsidiary-business unit (in layer 1).

Catastrophic failures in the Burkholz model

Observed effects (two layers with N_v = 1000 nodes each)

- A sudden jump in the number of finally failed nodes b_t when the initial number of failed nodes b_i is increased by just one
  - Tested for c_{10} = 0.5
  - b_i = 35 ⇒ b_t = 551
  - b_i = 36 ⇒ b_t = 1919 (an increase of 1368)
- A fast increase in the maximum Δb_t for Δb_i = 1 observed around c_{10} = 0.5
- See Fig. 2 (in the next column)

Catastrophic failures in the Burkholz model (cont.)

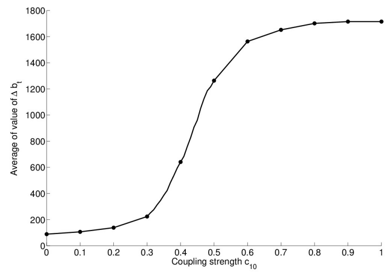

Fig. 2. The average from 30 runs of the biggest increase Δb_t (corresponding to increasing b_i by one) plotted against the coupling strength c_{10}.

Optimization in the Burkholz model

- Thresholds represent the ability of nodes to withstand loads caused by failures
- Increasing the threshold costs money (e.g. reserves that have to be created)
- Optimization of the thresholds
  - Fixed initial graph state (failed nodes)
  - The coupling strength c_{10} varied between 0.0 and 1.0
  - For each value of c_{10} the thresholds were optimized using the MOEA/D algorithm
  - Objectives: f_1 = cost (sum of thresholds), f_2 = the biggest observed Δb_t
- Solutions that increase the cost by at most c^* = 50 and c^* = 200 (5% and 20% of the original value) selected
- Optimized solutions have lower Δb_t corresponding to Δb_i = 1
- Sudden increase in the value of Δb_t occurs at higher c_{10}

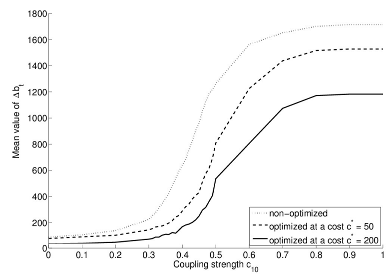

Fig. 3. The means from 30 runs of the maximum increase in the number of failed nodes Δb_t for the non-optimized system and using optimized solutions for c^* = 50 and 200

Results and conclusion

- The first experimental round shown that catastrophic failures are likely to occur in the studied system when the coupling strength c_{10} is sufficiently large
- To counteract these phenomena an EA was used to optimize the thresholds that represent the maximum load each business unit can take
- At a moderate cost the system can be made more resistant to failures
  - the magnitude of the cascading failures is diminished
  - stronger coupling may be present without triggering catastrophic failures

References

[1] B. Hartnell, "*Firefighter! An application of domination*", in: 20th Conference on Numerical Mathematics and Computing (1995).

[2] K. Michalak „*Auto-adaptation of Genetic Operators for Multi-objective Optimization in the Firefighter Problem*", Intelligent Data Engineering and Automated Learning IDEAL 2014, Lecture Notes in Computer Science, vol. 8669, pp. 484-491, Springer (2014).

[3] R. Burkholz, M. Leduc, A. Garas, F. Schweitzer „*Systemic risk in multiplex networks with asymmetric coupling and threshold feedback*", Physica D, 323-324, pp. 64-72 (2016).