The MOEA/D Algorithm with Gaussian Neighbourhoods for the Multiobjective Travelling Salesman Problem

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Introduction

Overview of the MOEA/D algorithm

- A decomposition-based algorithm introduced by Zhang and Li in 2007[1]
- A multiobjective optimization problem transformed into a set of single-objective problems using a set of weight vectors. Each specimen is a solution of a different subproblem.
- Tchebycheff scalarization (used in this work):

$$\text{minimize} \quad g^{te}(x|\lambda, z^*) = \max_{1 \le j \le m} \{\lambda_j |f_j(x) - z_j^*|\}$$
$$\text{subject to} \quad x \in \Omega,$$

where: λ - the weight vector assigned to the subproblem, z^* - the reference point

- The neighbourhood $B(i)$ of the specimen solving the subproblem scalarized using λ_i
  - T specimens associated with weight vectors closest to λ_i
  - Used for parent selection and solution replacement

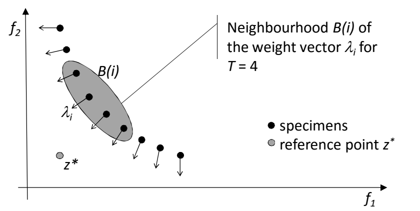

Fig. 1. An overview of concepts involved in the working of the MOEA/D algorithm in the case of minimization in a bi-objective problem.

Improvements in the 2009 version[2]

- The max number of solutions that can be replaced by a child solution limited to n_r
- Parents selected from the neighbourhood with probability δ (and from the entire population with probability $(1 - \delta)$. Typical value: $\delta = 0.9$

The effects of the MOEA/D parameterization

- The T parameter influences both the parent selection and solution replacement (parent selection and solution replacement cannot be parameterized separately)
- Parent selection probability changes abruptly between the neighbourhood and the general population
- Even values of T can cause the algorithm to converge asymmetrically (this problem is addressed in a separate paper[3])

The MOEA/D-G Algorithm

Highlights

- Modified parent selection scheme
- Parent selection controlled by a Gaussian distribution in the subproblem space
- The Gaussian distribution parameterized by the standard deviation σ

Parent selection in the MOEA/D-G

- For two objectives the weight vectors can be ordered in one dimension
- Determining which specimen to choose as a parent for the j-th subproblem
  - Draw a real number from the Gaussian distribution $N(0, \sigma)$
  - Round this number to the nearest integer n
  - Add n to j to get the index of the parent specimen $n + j$
- For $m > 2$ objectives weight vectors can be ordered in m - 1 dimensions
  - Draw an $(m - 1)$-dimensional real vector from the Gaussian distribution $N(0, \Sigma)$
  - Round the coordinates to the nearest integers
  - Add the obtained vector to the position of the subproblem to locate the parent

The MOEA/D-G Algorithm (cont.)

The effects of the introduced changes

- Separate parameterization of parent selection (using the σ parameter) and solution replacement (using the T and n_r parameters)
- Parent selection probability changing smoothly with the distance between weight vectors

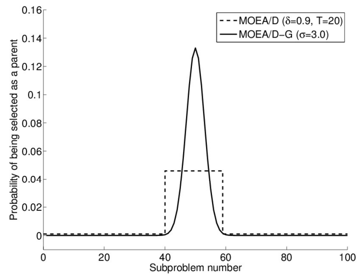

Fig. 2. Probabilities of selecting neighbours as parents for the subproblem located in the center of the graph for population size $N_{pop} = 100$. MOEA/D with $\delta = 0.9$ and $T = 20$. MOEA/D-G with $\sigma = 3.0$

Experiments and Results

Test problems and parameters

- Biobjective TSP instances kroAB*nnn*, with *nnn* = 100, . . ., 750[4]
- Three sets of T and n_r values ($T = 5$, $n_r = 2$), ($T = 20$, $n_r = 2$), ($T = 40$, $n_r = 4$)
- The probability that parent solutions are selected from the neighbourhood (in MOEA/D only): $\delta = 0.9$
- Five values of standard deviation in MOEA/D-G: σ = 1.0, 2.0, 3.0, 4.0, 5.0

Results

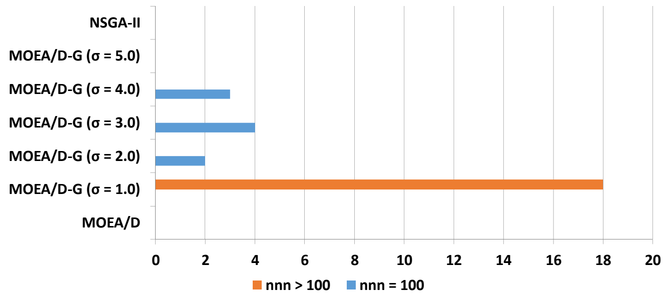

Fig. 3. The number of times each of the methods produced the best result for a given test instance

Conclusion

- MOEA/D-G has a modified parent selection scheme using Gaussian distribution
- Tested on biobjective instances of the TSP with the number of cities from 100 to 750
- Outperforming both comparison methods: the NSGA-II and the original MOEA/D
- Best values of the standard deviation σ in the MOEA/D-G: for *nnn* = 100 the $\sigma \in [2, 4]$ and for n > 100 the $\sigma = 1.0$ were the best choices

References

[1] Q. Zhang, H. Li, "*MOEA/D: A multiobjective evolutionary algorithm based on decomposition*", IEEE Trans. Evol. Comput. 11, pp. 712–731 (2007).

[2] H. Li, Q. Zhang, "*Multiobjective optimization problems with complicated Pareto sets, MOEA/D and NSGA-II*", IEEE Trans. Evol. Comput. 13, pp. 284–302 (2009).

[3] K. Michalak, "*The effects of asymmetric neighborhood assignment in the MOEA/D algorithm*", Applied Soft Computing 25, pp. 97–106 (2014).

[4] T. Lust, "*Research Interests > Multiobjective TSP*", online, accessed 2017.07.14, URL: https://sites.google.com/site/thibautlust/research/multiobjective-tsp