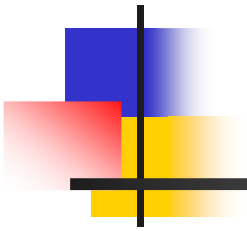

Local Search Based on a Local Utopia Point for the Multiobjective Travelling Salesman Problem

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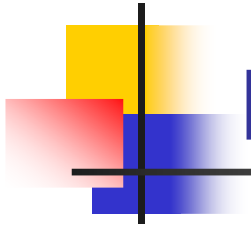


Presentation Plan

- TSP and multiobjective TSP
- Overview of the algorithm
- Local search procedures (ULS and PLS)
- Evolutionary algorithm – Local search trade-off
- Experiments
- Results
- Conclusion

TSP and multiobjective TSP

- Travelling Salesman Problem
  - Find the shortest route connecting n cities
  - Formally, for a given $n \times n$ cost matrix $C = [c_{ij}]$, $i, j \in \{1, \ldots, n\}$ minimize:

$$f(p) = c_{p(n)p(1)} + \sum_{i=1}^{n-1} c_{p(i)p(i+1)}$$

    subject to $p \in P_n$, where P_n - the set of all permutations of numbers $1, \ldots, n$

- Multiobjective TSP
  - Minimize m objective functions $f_j(p)$, $j = 1, \ldots, m$
  - Each $f_j(p)$ is calculated using a different cost matrix C_j
  - Real life criteria example: time and money

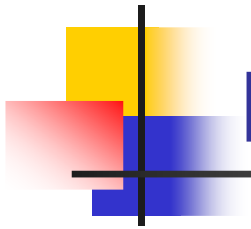
Overview of the Algorithm

- NSGA-II with a local search
- Genotype = permutation
- Operator: inver-over

```
select (randomly) a city c from S'
repeat
{
  if (rand() ≤ p)
    select the city c' from the remaining cities in S'
  else
  {
    select (randomly) an individual from P
    assign to c' the 'next' city to the city c in the selected individual
  }
  if (the next city or the previous city of city c in S' is c')
    exit from repeat loop
  inverse the section from the next city of city c to the city c' in S'
  c = c'
}
```

Source:

Tao, G., Michalewicz, Z. „Inver-over Operator for the TSP", Parallel Problem Solving from Nature PPSN V, Lecture Notes in Computer Science Volume 1498, 1998, pp 803-812



Local search procedures

- ULS

  Local Search Based on a Local Utopia Point

- PLS

  Pareto-based Local Search

Local search is used for improving new specimens (in a new population and those generated by genetic operators)

Local search procedures

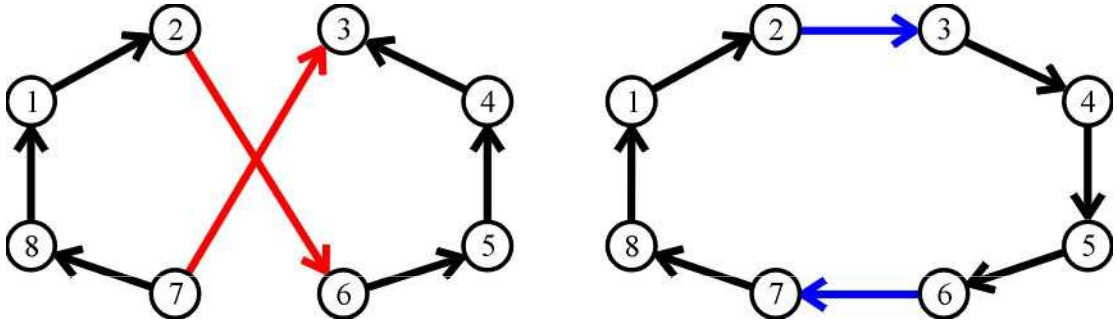
- Both ULS and PLS are best-improvement procedures
- Best-improvement local search loop:

```
repeat
    improved := false
    for x' ∈ N(x) do
        if x' is better than x then
            x := x'
            improved := true
        end if
    end for
until (not improved)
```

- Various neighbourhood generation procedures and acceptance criteria are possible

Local search procedures

- Both ULS and PLS use neighbourhoods generated by the 2-opt operator



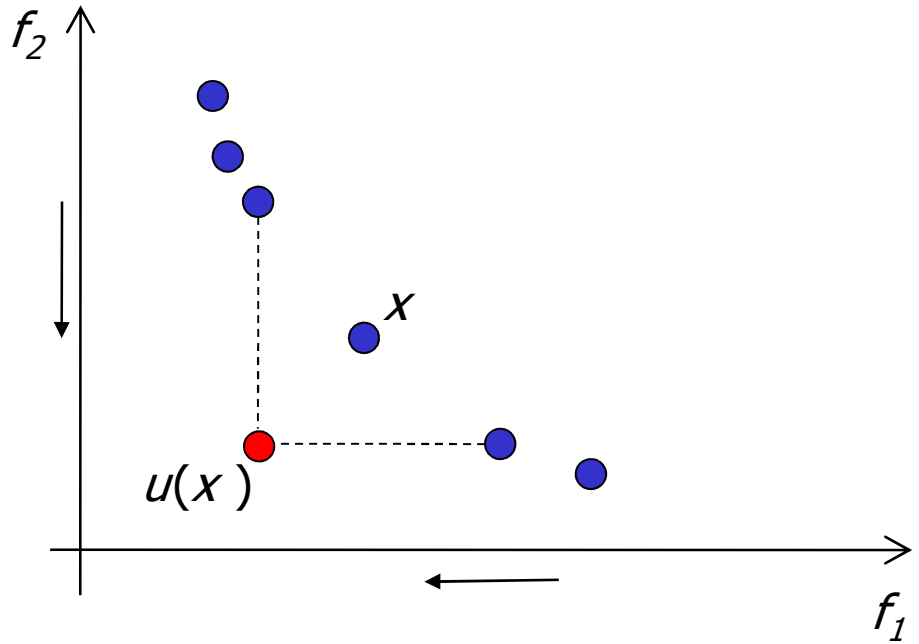
Source: http://www.devx.com/dotnet/Article/33574/0/page/3

- Different acceptance criteria are used
  - PLS: x' is accepted if it Pareto-dominates x (that is, is not worse w.r.t. all criteria and better w.r.t. at least one)
  - ULS: x' is accepted if it is closer to a local utopia point $u(x)$ than x is

Local utopia point in the ULS

- A utopia point $u(x)$ is generated for a given x
- All coordinates set to maximum values of objectives in the population P less than those of x
- Formally, each coordinate $u(x)_i$ is set to:

$$u(x)_i = max\{f_i(x') : x' \in P - \{x\} \wedge f_i(x') < f_i(x)\}$$



The ULS procedure

IN: d - the dimensionality of the objective space
x - the solution to improve
P - current population, $x \in P$

OUT: x - the solution improved by the local search

```
for i = 1,...,d do                                  # 1. Establish a local utopia point
    u(x)_i = MIN_VALUE
    for x' ∈ P − {x} do
        if f_i(x') < f_i(x) and f_i(x') > u(x)_i then
            u(x)_i = f_i(x')
        end if
    end for
end for

repeat                                              # 2. Improve the solution
    improved := false
    for x' ∈ N(x) do
        if d(F(x'), u(x)) < d(F(x), u(x)) then      # 2.1. Check if the new solution
            x := x'                                 #      is better than the old one
            improved := true
        end if
    end for
until (not improved)

return x
```

The PLS procedure

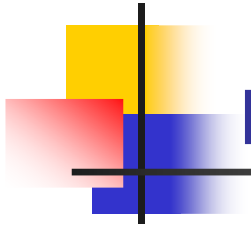
IN: d - the dimensionality of the objective space
x - the solution to improve
P - current population, $x \in P$

OUT: x - the solution improved by the local search

~~for $i = 1, \ldots, d$ do~~ # 1. Establish a local utopia point
~~$u(x)_i = \text{MIN\_VALUE}$~~
~~for $x' \in P - \{x\}$ do~~
~~if $f_i(x') < f_i(x)$ and $f_i(x') > u(x)_i$ then~~
~~$u(x)_i = f_i(x')$~~
~~end if~~
~~end for~~
~~end for~~

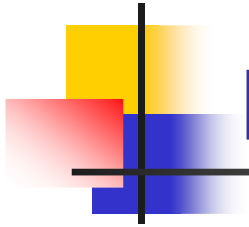
repeat # 2. Improve the solution
improved := false
for $x' \in N(x)$ do
if $x' \succ x$ then # 2.1. Check if the new solution
x := x' # is better than the old one
improved := true
end if
end for
until (not improved)

return x



Evolutionary algorithm – Local search trade-off

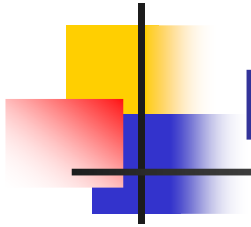
- Computational resources can be assigned to EA and local search in various proportions
- Several strategies tested:
  - PLS(P = 1.0) – local search performed for each specimen
  - PLS(P = 0.5) – local search performed for each specimen with probability ½
  - PLS(asc) – probability of performing the local search increasing from 0.0 to 1.0 with time
  - PLS(desc) – probability of performing the local search decreasing from 1.0 to 0.0 with time
  - PLS(none) – no local search (100% resources assigned to the EA)



Experiments

- Test instances kroAB*nnn*, for *nnn* = 100, 150, 200, 300, 400, 500, 750 and 1000[1]
- Population size: 100
- Running time: 300 seconds
- Random inverse rate: $\eta = 0.02$
- 30 repetitions for each test instance and algorithm

[1] Thibaut Lust: Multiobjective TSP (2015).
https://sites.google.com/site/thibautlust/research/multiobjective-tsp
Accessed 29 January 2015



Results

- Median values for hypervolume and IGD calculated from 30 runs
- Statistical test performed with the hypothesis that ULS and a compared method produce the same median results
- Interpretation:
  - \+ better median for ULS, p-value ≤ 0.05
  - \# better median for ULS, p-value > 0.05
  - \- worse median for ULS
- Family-wise error:
  - \+ family-wise probability of type I error ≤ 0.05
  - \# family-wise probability of type I error > 0.05
  - \- ULS produced worse result for one of the test instances

Results

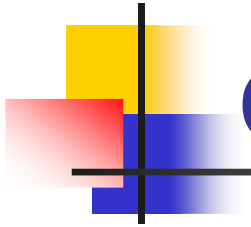
Table 9. Results of the statistical analysis comparing hypervolume and IGD values obtained using the ULS method with other tested methods.

N	Hypervolume					IGD				
	none	PLS ($P = 0.5$)	PLS ($P = 1.0$)	PLS (asc)	PLS (desc)	none	PLS ($P = 0.5$)	PLS ($P = 1.0$)	PLS (asc)	PLS (desc)
100	+	+	+	+	+	+	+	+	+	+
150	+	+	+	+	+	+	+	+	+	+
200	+	+	+	+	+	+	+	+	+	+
300	+	+	+	+	+	+	+	+	+	+
400	+	+	+	+	+	+	+	+	+	+
500	+	+	+	+	+	+	+	+	+	+
750	+	+	+	+	+	+	+	+	+	+
1000	+	+	+	+	+	+	+	+	+	+
F-W	+	+	+	+	+	+	+	+	+	+

Results

Table 10. Results of the statistical analysis comparing the results obtained using the PLS (desc) method with other tested methods.

N	Hypervolume					IGD				
	none	PLS ($P = 0.5$)	PLS ($P = 1.0$)	PLS (asc)	ULS	none	PLS ($P = 0.5$)	PLS ($P = 1.0$)	PLS (asc)	ULS
100	+	+	–	+	–	+	+	–	+	–
150	+	+	–	+	–	+	+	–	+	–
200	+	+	–	+	–	+	+	–	+	–
300	+	+	+	+	–	+	+	#	+	–
400	+	+	+	+	–	+	+	+	+	–
500	+	+	+	+	–	+	+	+	+	–
750	+	+	+	+	–	+	#	+	+	–
1000	+	+	+	+	–	+	+	+	+	–
F-W	+	+	–	+	–	+	#	–	+	–



Conclusion

- A local search method based on a local utopia point (ULS) was proposed
- ULS was compared to a Pareto-based local search (PLS) on several instances of the multiobjective TSP
- For the Pareto-based method five different strategies of invoking the local search procedure were tested
- Best PLS performs the local search with probability P = 1.0 at the beginning of the optimization and the reduces this probability linearly to P = 0.0
- ULS outperformed all Pareto-based methods

Thank you!

(questions?)