Reducing Systemic Risk in Multiplex Networks Using Evolutionary Optimization

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Spreading of threats in graphs

Cascading failures

- A system of interconnected entities
- Some of these entities initially fail and these failures overload other entities that also fail
- Examples: power grid failures, a wave of bankruptcies

Coupling strength and phase transitions

- The final number of failures depends on the coupling strength $c$ (how much the failure of one node affects the other nodes)
- For lower $c$, the failures quickly stop spreading, for higher $c$, the failures may affect (almost) all the nodes (catastrophic failure)
- An abrupt change in the behaviour (phase transition) may occur at a critical value of the coupling strength

The Burkholz model[3]

- A two-layer network of companies. Layer 0: core units, layer 1: subsidiary units.
- Connections in each layer: Erdős-Rényi graph (here with $N_v = 1000$ and $p_s = 0.003$)
- Some nodes initially fail. The failures affect the neighbouring nodes
  - the load depends on the fraction of the neighbours that have failed
  - thresholds (drawn from $U(0,1)$) determine how much load each unit can withstand
- Asymmetric coupling between layers
  - $c_{01} = 1$: core unit failure induces an instant failure of the subsidiary unit
  - $c_{01} < 1$: subsidiary unit failure exerts a load on the core unit (lowers its threshold by multiplying by $1 - c_{01}$)

Catastrophic failures in the Burkholz model

Observed effects (two layers with $N_v = 1000$ nodes each)

- A sudden jump in the number of finally failed nodes $b_t$ when the initial number of failed nodes $b_i$ is increased by just one
  - Tested for $c_{01} = 0.5$
  - $b_i = 35 \Rightarrow b_t = 551$
  - $b_i = 36 \Rightarrow b_t = 1919$ (an increase of 1368)
- A fast increase in the maximum $\Delta b_t$ for $\Delta b_i = 1$ observed around $c_{01} = 0.5$
- See Fig. 2 (in the next column)

Optimization in the Burkholz model

- Thresholds represent the ability of nodes to withstand loads caused by failures
- Increasing the threshold costs money (e.g. reserves that have to be created)
- Optimization of the thresholds
  - Fixed initial graph state (failed nodes)
  - The coupling strength $c_{01}$ varied between 0.0 and 1.0
  - For each value of $c_{01}$, the thresholds were optimized using the MOEA/D algorithm
  - Objectives: $f_1 = \text{cost (sum of thresholds)}, f_2 = \text{the biggest observed } \Delta b_t$
- Solutions that increase the cost by at most $c^* = 50$ and $c^* = 200$ (5% and 20% of the original value) selected
- Optimized solutions have lower $\Delta b_t$ corresponding to $c_{01} = 1$
- Sudden increase in the value of $\Delta b_t$ occurs at higher $c_{01}$

Results and conclusion

- The first experimental round shown that catastrophic failures are likely to occur in the studied system when the coupling strength $c_{01}$ is sufficiently large.
- To counteract these phenomena an EA was used to optimize the thresholds that represent the maximum load each business unit can take.
- At a moderate cost the system can be made more resistant to failures
  - the magnitude of the cascading failures is diminished
  - stronger coupling may be present without triggering catastrophic failures

References