Reducing systemic risk in multiplex networks using evolutionary optimization

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ABSTRACT

This paper concerns the topic of cascading failures in a system containing economic entities. Cascading failures occur when some of the entities fail and, because of the relationships on the market, cause many more entities to fail. In this paper a multiobjective optimization problem is formulated for an economic model involving companies composed of a core-business and a subsidiary-business unit. A multiobjective optimization algorithm is used to solve the optimization problem, thereby making the system more robust against failures. Subsequently, the properties of the optimized system are studied. Solving the proposed optimization problem diminishes the severity of cascading failures occurring in the system and increases the threshold coupling strength between business units for which the cascading failure occurs.

CCS CONCEPTS

•Mathematics of computing → Evolutionary algorithms; **•Applied computing → Multi-criterion optimization and decision-making;**

KEYWORDS

Graph-based optimization, Multi-objective evolutionary optimization, Cascading failures

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1 INTRODUCTION

The Burkholz model. In this paper cascading failures in an economic system are studied using a model proposed by Burkholz et al. [1] which represents interdependencies between companies composed of a core-business and a subsidiary-business unit. The model is a multiplex network consisting of two graphs (layers) both with the same number of nodes N_v in which nodes represent core-business units (layer 0) and subsidiary-business units (layer 1). Connections within layers represent interactions between companies operating in the same market sector and connections between layers represent interactions between the core and subsidiary units of the same company.

Cascading failures in the Burkholz model. The Burkholz model is used for studying the spreading of bankruptcies in the system composed of many interconnected business units. Each unit $v_j^{(l)}$ (where $j = 1, \ldots, N_v$, $l \in \{0, 1\}$) has a threshold $\Theta_j^{(l)} \in [0, 1]$ which represents its ability to withstand the load exerted by failures of neighbouring units. Initially, some of the units fail and if the fraction of the failed neighbours located in the same layer exceeds or matches the threshold $\Theta_j^{(l)}$ of any still operating unit $v_j^{(l)}$, this unit fails also. Moreover, when a subsidiary unit $v_j^{(1)}$ fails it strains the core unit $v_j^{(0)}$ by lowering its threshold $\Theta_j^{(0)}$ multiplying it by $1 - c_{10}$, where $c_{10} \in [0, 1]$. On the other hand when the core unit $v_j^{(0)}$ fails the subsidiary unit $v_j^{(1)}$ inevitably fails also (thus $c_{01} = 1$).

In preliminary experiments cascading failures were studied in a Burkholz model consisting of two Erdős-Renyi (E-R) graphs $G(N_v, P_e)$ with the number of vertices $N_v = 1000$ and the probability of connecting any two vertices with an edge $P_e = 3.0/N_v = 0.003$. The values of $\Theta_j^{(l)}$ were uniformly drawn from $[0, 1]$ and the coupling strength c_{10} was set to 0.5. The total number of failed nodes b_t was observed as a function of the number of initially failed nodes b_i. A sudden increase from $b_t = 551$ to $b_t = 1919$ was observed for an increase in b_i of *just one* (from $b_i = 35$ to $b_i = 36$).

The influence of the coupling strength. For a fixed set of thresholds $\Theta_j^{(l)}$ the influence of the coupling strength c_{10} can be observed in the following manner. First, two permutations $\pi^{(0)}$ and $\pi^{(1)}$ of N_v elements are randomly generated. Then, c_{10} is varied from 0 to 1 and for each value of c_{10} the number of initially failed companies b_i is increased from 1 to N_v and b_i companies pointed to by the first b_i elements in $\pi^{(l)}$ go bankrupt in each layer l. Then, a simulation is run and a corresponding number of the failed nodes at the end of the simulation $b_t(b_i)$ is recorded. The influence of the coupling strength can be measured by $\Delta b_t = \max_{b_i}(b_t(b_i + 1) - b_t(b_i))$ which is the largest increase in the total number of failed nodes when the number of initially failed nodes is increased by one.

2 SYSTEM OPTIMIZATION

In this paper the model proposed by Burkholz is extended by allowing the companies to be strengthened against bankruptcy by adjusting the thresholds $\Theta_j^{(l)}$.

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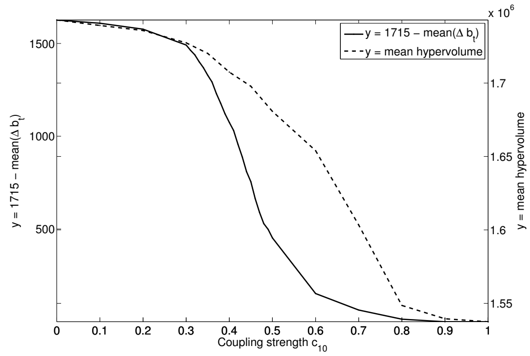

Figure 1: The means from 30 runs of the maximum increase in the number of failed nodes Δb_t (solid line) and the hypervolume attained in 300 generations (dashed line)

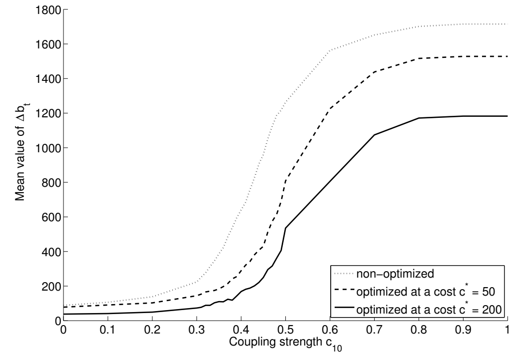

Figure 2: The means from 30 runs of the maximum increase in the number of failed nodes Δb_t for the non-optimized system and using optimized solutions for $c^* = 50$ and 200

The optimization problem. In this paper a multiobjective optimization problem is formulated in which the search space is $\Omega = [0,1]^{2N_v}$ (each threshold $\Theta_j^{(l)}$ can be adjusted separately). The first objective is the total cost of protection against bankruptcies:

$$f_1 = \sum_{j=1,\ldots,N_v,\, l \in \{0,1\}} \Theta_j^{(l)} \tag{1}$$

and is to be minimized. The second objective is determined by simulating the spreading of bankruptcies starting with a fixed set of failed units containing b_i units in each layer. The value of f_2 is the number of failed units when the cascading failure stops spreading.

Evolutionary optimization. The optimization was performed using the MOEA/D algorithm [2] with the population size $N_{pop} = 100$ and the number of generations $N_{gen} = 300$. The hypervolume indicator was used for comparing Pareto fronts obtained for different values of c_{10}. The tests were performed on 30 test instances each containing a pair of graphs generated as described in Section 1 with $b_i = 100$ nodes initially failed in each layer.

In Figure 1 the mean value of the hypervolume attained after 300 generations is plotted against the coupling strength c_{10} (dashed line). In the same figure the mean $\overline{\Delta b_t}$ of the biggest increases Δb_t (corresponding to increasing b_i by one) is shown flipped vertically by plotting a value of $1715 - \overline{\Delta b_t}$ (solid line). Both means are calculated over 30 runs of the test. It can be observed that with the growing value of c_{10} saving the companies while minimizing costs becomes more difficult and the values of the hypervolume drop. Comparison of $\overline{\Delta b_t}$ and the hypervolume attained by the optimizer for the same values of c_{10} presented in Figure 1 suggests that when the mean magnitude of cascading failures gets larger it is more difficult to find good trade-offs between protecting the companies and minimizing the costs incurred by making the system more robust.

Cascading failures in the optimized system. In order to observe how the optimized system reacts to changes in the coupling strength c_{10}, the thresholds were set to values taken from solutions not exceeding the total sum of thresholds in the non-optimized system by more than a preset value c^* and values of Δb_t were calculated. Mean values of Δb_t observed in 30 runs for $c^* = 50$ and 200 are plotted in Figure 2 along with mean values observed for the original, randomly selected thresholds for comparison. Note, that the costs of $c^* = 50$ and 200 are not particularly high. For the network composed of two graphs with $N_v = 1000$ nodes each and thresholds drawn uniformly from $[0,1]$ the expected value of the sum of the thresholds is 1000, so these costs represent an increase of 5% and 20% respectively.

From the plots presented in Figure 2 it can be reasoned that optimizing thresholds in the network can reduce the magnitude of the cascading failures somewhat. Even for $c_{10} > 0.8$ when the mean size of the cascading failure stays more or less the same the values observed for $c^* = 50$ are about 10% lower than for the non-optimized system. For $c^* = 200$ the mean size of the cascading failure is reduced by about 30%. Also, the appearance of cascading failures seems to be shifted towards higher values of c_{10} as evidenced by the relative positions of the steepest parts of the plots. For the tested values of the costs c^* the cascading effect does not disappear entirely, however, for $c^* = 200$ its magnitude seems to be diminished and it occurs at higher values of the coupling strength c_{10}.

3 CONCLUSION

In this paper a multiobjective optimization problem was formulated aiming at making an economic system more resistant to cascading failures. It was observed that when sufficiently large additional costs can be borne it is possible to diminish the magnitude of the cascading failures occurring in the system and allow stronger couplings between network layers without triggering such events.

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