The MOEA/D Algorithm with Gaussian Neighbourhoods for the Multiobjective Travelling Salesman Problem

Krzysztof Michalak
Department of Information Technologies, Institute of Business Informatics, Wroclaw University of Economics
Komandorska 118-120
Wroclaw, Poland 53-345
krzysztof.michalak@ue.wroc.pl

ABSTRACT

In this paper the MOEA/D-G algorithm is proposed which is a modification of the MOEA/D algorithm using Gaussian distributions to determine the probability with which neighbours of a given subproblem are selected as parents of new specimens assigned to this subproblem. The proposed method is applied to the Multiobjective Travelling Salesman Problem (MOTSP). Solutions found by the MOEA/D-G algorithm have a better quality than those found by the original MOEA/D version. Also, given equal computation time, all versions of the MOEA/D outperform the NSGA-II algorithm.

CCS CONCEPTS

•**Mathematics of computing → Combinatorial optimization;** *Evolutionary algorithms;* •**Applied computing →** *Multi-criterion optimization and decision-making;*

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1 INTRODUCTION

The Multiobjective Evolutionary Algorithm based on Decomposition (MOEA/D) [2] uses a set of weight vectors $\{\lambda^{(j)}\}_{j=1,\ldots,N_{pop}}$ (where N_{pop} is the population size) to decompose a multiobjective optimization problem to a set of single objective problems. A neighbourhood structure is defined on these subproblems and parent selection for the j-th subproblem is performed with a probability δ from its neighbourhood $B(j)$ containing T specimens, and with a probability $1 - \delta$ from the entire population. Because a typical value of the δ parameter is $\delta = 0.9$ this mechanism prefers parent selection from "similar" subproblems, but also allows exchanging information with the general population. In the original MOEA/D the probability of selecting a given specimen as a parent changes abruptly between the T neighbours and the rest of the population. In the MOEA/D-G algorithm the parent selection mechanism uses a Gaussian distribution for determining the probability of selecting a particular specimen.

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2 THE MOEA/D-G ALGORITHM

In the MOEA/D-G algorithm parents are selected from the entire population with a probability determined by a Gaussian distribution. Parent selection in a biobjective case is performed as presented in Algorithm 1. The gsamp() function draws a random number from a Gaussian distribution with given parameters. The round() function rounds a given real number to the nearest integer. In a multiobjective case with $m > 2$ objectives this selection procedure can be performed by drawing a vector of $m - 1$ real numbers from an $(m - 1)$-dimensional Gaussian distribution and by adding the rounded values to coordinates of the subproblem for which to select parents on a $m - 1$ dimensional grid of weight vectors.

Algorithm 1 Procedure used for selecting parents for the j-th subproblem.

IN: j - the number of the subproblem
σ - the standard deviation of the Gaussian distribution
N_s - the number of parents to select
OUT: S - the set of indices of the selected parents
$S := \emptyset$
while $|S| < N_s$ **do**
  do
    $n := \text{round}(\text{gsamp}(0, \sigma))$
    $n := n + j$
  while $((n < 1)$ **or** $(n > N_{pop})$ **or** $(n \in S))$
  $S := S \cup \{n\}$
end while

In the MOEA/D the T parameter influences both parent selection (additionally parameterized by the probability δ) and solution replacement (additionally parameterized by n_r - the maximum number of replaced specimens). In the MOEA/D-G the neighbourhood of size T is used for solution replacement in exactly the same way as in the original MOEA/D, so both the T and the n_r parameters are used in this process. On the other hand the parent selection procedure in the MOEA/D-G does not use the T nor δ parameters, because the standard deviation σ is used instead. Therefore, in the MOEA/D-G the parameters T and n_r control solution replacement and σ controls parent selection, so these two aspects can be parameterized independently as opposed to the MOEA/D.

When the parent selection procedure proposed in this paper is used, the probability of selecting a certain specimen as a parent changes smoothly with the distance between subproblems. In Figure 1 the distribution used by the original MOEA/D is presented along with the distribution used by MOEA/D-G. Clearly, MOEA/D-G uses higher probabilities than MOEA/D for selecting parents

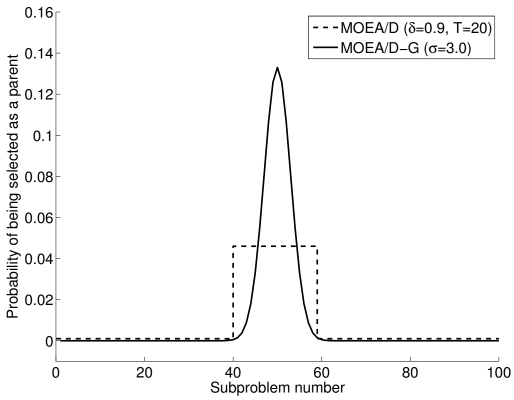

Figure 1: Probabilities of selecting neighbours as parents for a subproblem located in the center of the graph for population size $N_{pop} = 100$. MOEA/D with parameters $\delta = 0.9$ and $T = 20$ and MOEA/D-G with $\sigma = 3.0$.

from nearby subproblems and allows selecting distant specimens, but with low probabilities.

3 EXPERIMENTS AND RESULTS

The proposed algorithm was tested on a suite of kro⟨N⟩ instances of the Multiobjective Travelling Salesman Problem (MOTSP) [1] with the number of cities $N = 100, \ldots, 750$. In this problem each solution is a permutation of numbers $1, \ldots, N$ which represents the order in which N cities are visited. The results obtained using the MOEA/D-G were compared to those obtained using the standard version of the MOEA/D algorithm and the NSGA-II algorithm. All the algorithms used the Inver-Over operator [3] and a 2-opt local search [4]. The results were compared with respect to computation time: 600s for $N = 100$, 1 800s for $N = 150$, 3 600s for $N = 200$, 10 800s for $N = 300$, 32 400s for $N = 400$, 72 000s for $N = 500$ and 345 600s for $N = 750$. The population size was $N_{pop} = 100$. To assess the influence of the σ parameter on the quality of the results, five different values 1.0, 2.0, 3.0, 4.0, and 5.0 were tested. Also, three different pairs of values were assigned to the neighbourhood size T and n_r - the maximum number of solutions replaced by a child solution, namely ($T = 5, n_r = 2$), ($T = 20, n_r = 2$) and ($T = 40, n_r = 4$). In the experiments a value of $\delta = 0.9$ was used for the MOEA/D algorithm. Pareto fronts produced by the tested algorithms were compared using the hypervolume indicator (HV) introduced in [5]. During the tests each algorithm with each parameter set was run on each test instance 30 times and the median HV from these 30 runs was calculated. The number of times the best result was obtained by MOEA/D-G with $\sigma = 1.0, \ldots, 5.0$ was 18, 2, 4, 3 and 0 respectively. The MOEA/D and the NSGA-II never produced the best result. Table 1 shows the minimum, average and maximum ratio of median hypervolumes attained by the algorithms.

From the presented results it is clear, that the MOEA/D-G algorithm outperforms both the original MOEA/D and the NSGA-II. To verify the hypothesis that the observed advantage is statistically significant, statistical testing was performed using the Wilcoxon

Table 1: Min, avg and max ratio of median hypervolumes.

	$HV_{\text{MOEA/D}}/HV_{\text{NSGA-II}}$	$HV_{\text{MOEA/D-G}}/HV_{\text{MOEA/D}}$				
		$\sigma = 1.0$	$\sigma = 2.0$	$\sigma = 3.0$	$\sigma = 4.0$	$\sigma = 5.0$
Min	1.09	1.03	1.03	1.04	1.03	1.03
Avg	2.31	1.12	1.11	1.10	1.10	1.09
Max	4.10	1.22	1.20	1.19	1.18	1.16

rank test. The null hypothesis in this test was the equality of median results obtained by each MOEA/D-G variant (e.g. MOEA/D-G ($\sigma = 1.0$)) and a comparison method (MOEA/D, NSGA-II). In every case the obtained p-value was less than $2 \cdot 10^{-6}$. Such low p-values confirm that the differences in median values obtained by the algorithms are significant and so is the observed advantage of the MOEA/D-G over the MOEA/D and NSGA-II. Given the very low p-values obtained in all individual cases the family-wise corrected value is also well below 0.01. The relationship between the standard deviation σ of the Gaussian distribution and the quality of the results depends on the problem instance size. For $N = 100$ cities the best result was obtained for $\sigma \in [2, 4]$. For larger instances the best result was always obtained for $\sigma = 1.0$ for which it is the most probable to select parents from nearby subproblems. On the other hand for all test instances except kroAB100 the best result was achieved for $T = 40$. This may indicate the fact that incorporating parts of solutions found for distant subproblems benefits the search process, but the probability of doing so must be decreased with the distance, not kept constant as in the original MOEA/D.

4 CONCLUSION

In this paper a new variant of the MOEA/D algorithm is studied that uses a Gaussian distribution instead of a preset neighbourhood to select parents for producing new solutions for the subproblems. The proposed algorithm was tested on nine instances of the Multiobjective Travelling Salesman Problem. The MOEA/D-G algorithm performed better than the original version of the MOEA/D algorithm and the NSGA-II on all the tested problems with all parameters settings that were used in the experiments.

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