Auto-adaptation of Genetic Operators for Multi-objective Optimization in the Firefighter Problem

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Presentation Plan

- The Firefighter Problem
- The Evolutionary Algorithm
  - solution representation
  - operators
  - autoadaptation mechanism
- Experiments
- Results

The Firefighter Problem

- Introduced by Hartnell in 1995[1]
- Spread of fire is modelled on an undirected graph
  - discrete-time model
  - vertices are labeled:
    - 'B' – burning
    - 'D' – defended by firefighters
    - 'U' – untouched

    let $l(v)$ denote the label assigned to node v
  - initially a certain number of nodes are burning ('B') and the remaining ones are untouched

[1] Hartnell, B.: *Firefighter! An application of domination*. In: 20th Conference on Numerical Mathematics and Computing (1995)

The Firefighter Problem

- At each time step
  - N_f firefigters are assigned to the untouched ('U') nodes. These nodes become defended ('D')
  - nodes adjacent to the burning nodes catch on fire (unless they are defended by firefighters)
- The simulation stops when the fire is contained or when all undefended nodes are burning
- Task: Find the best assignment of firefighters to graph nodes

The Firefighter Problem

- Single-objecitve: save as many nodes of the graph as possible
- Multi-objective*)
  - for each node v there are m different values $v_i(v)$, $i = 1, \ldots, m$
  - The m objectives f_i, $i = 1, \ldots, m$ attained by a given solution are calculated as follows:

$$f_i = \sum_{v \in V : l(v) \neq 'B'} v_i(v)$$

  - where:

    $v_i(v)$ - the value of node v according to the i-th criterion

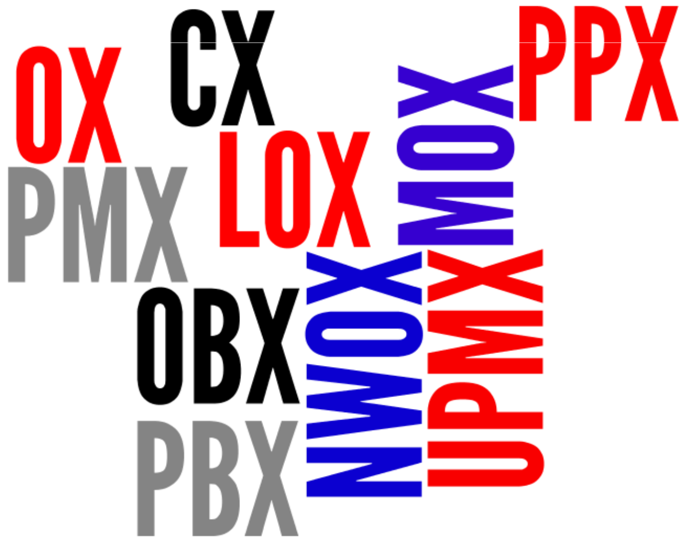
*) introduced in this paper

The Evolutionary Algorithm

- The algorithm is based on an NSGA-II algorithm
- Solution representation:
  - genotype = a permutation *P* of N_v numbers
  - permutation *P* represents the order in which firefighters are assigned to the nodes
  - at each time step the firefighters are assigned to the first N_f untouched ('U') nodes taken from the permutation *P*
- Objectives:
  - *m* objectives $f_1, \ldots, f_m$
  - *m*+1-th objective: the number of nodes saved (i.e. labelled either 'D' or 'U' at the end of the simulation)

Operators

- Typical operators used for permutation-based problems
- 10 crossover operators
  - CX, LOX, MOX, NWOX, OBX, OX, PBX, PMX, PPX and UPMX
- 5 mutation operators
  - displacement mutation
  - insertion mutation
  - inversion mutation
  - scramble mutation
  - transpose mutation



Autoadaptation

- Based on:
  - b_i - the number of times when a given operator produced an improved specimen
  - n_i - the number of times each operator was used
  - success rate $s_i = b_i / n_i$
- Probability assignment
  - each of the N_{op} operators is given a minimum probability P_{min}
  - the remaining $1 - N_{op} \cdot P_{min}$ is divided proportionally to
    - b_i (in the RawScore method)
    - s_i (in the SuccessRate method)
- The selection of crossover and mutation operators is performed separately

Experiments

- Test data = graphs with various density of edges
- Parameters
  - N_v = the number of vertices
  - P_{edge} = probability of adding an edge for a given pair of vertices
  - N_s = the number of vertices at which the fire is started (set to 1, $0.04 \cdot N_v$ and $0.1 \cdot N_v$)
  - N_f = The number of firefighters assigned at each time step (set to $N_f = 2 \cdot N_s$)

N_v	P_{edge}	N_s
50	0.05	1, 2 and 5
100	0.02	1, 4 and 10
500	0.0055	1, 20 and 50
1000	0.0025	1, 40 and 100

Experiments

- The density of the graph heavily impacts the progress of the simulation
  - low density = the fire is easily contained
  - high density = the fire is very hard to contain and all nodes burn except those protected by firefighters
- Preliminary round of tests performed to fine-tune the value of the P_{edge} parameter
- Solutions evaluated using the hypervolume indicator
- Averages taken from 10 runs

Results

N_v	P_{edge}	N_s	RawScore auto-adaptation	SuccessRate auto-adaptation
50	0.05	1	$4.6143 \cdot 10^6$	$\underline{5.2143 \cdot 10^6}$
		2	$5.5699 \cdot 10^7$	$\underline{5.6312 \cdot 10^7}$
		5	$5.2817 \cdot 10^7$	$\underline{5.4063 \cdot 10^7}$
100	0.02	1	$4.6310 \cdot 10^8$	$\underline{7.2057 \cdot 10^8}$
		4	$4.5708 \cdot 10^8$	$\underline{5.2746 \cdot 10^8}$
		10	$3.9548 \cdot 10^8$	$\underline{5.1009 \cdot 10^8}$
500	0.0055	1	$2.0798 \cdot 10^7$	$\underline{2.1406 \cdot 10^7}$
		20	$3.3457 \cdot 10^9$	$\underline{3.5775 \cdot 10^9}$
		50	$\underline{1.1501 \cdot 10^{10}}$	$0.9089 \cdot 10^{10}$
1000	0.0025	1	$\underline{1.0167 \cdot 10^8}$	$1.0016 \cdot 10^8$
		40	$\underline{2.7972 \cdot 10^{10}}$	$2.6013 \cdot 10^{10}$
		100	$\underline{1.1303 \cdot 10^{11}}$	$1.0949 \cdot 10^{11}$

Hypervolume values attained by both auto-adaptation methods

Results

N_v	P_{edge}	N_s	CX	LOX	MOX	NWOX	OBX	OX	PBX	PMX	PPX	UPMX
50	0.05	1	339.7	172.9	312.5	210.7	272.7	148.0	182.6	167.9	140.4	231.3
		2	703.9	232.9	159.5	252.4	210.5	121.5	126.7	340.6	335.0	326.6
		5	197.9	120.6	598.7	227.2	215.6	270.1	205.8	112.2	149.5	93.6
100	0.02	1	770.7	314.1	269.7	532.3	248.8	106.4	609.1	475.4	447.0	258.8
		4	1112.6	949.7	1107.3	1440.8	1160.9	458.6	871.5	801.8	643.8	449.1
		10	917.8	1361.1	605.0	828.8	620.6	263.2	1084.6	1267.2	847.1	392.4
500	0.0055	1	4905.4	2907.4	4662.4	3410.5	6970.6	1293.1	8531.8	3733.0	8706.2	4891.3
		20	20910.5	12966.6	25450.5	15624.4	17796.1	1330.2	29291.9	7203.9	24431.8	2608.8
		50	28899.5	15642.5	14174.3	15757.6	26394.8	2181.3	22251.9	6649.2	11675.2	1408.3
1000	0.0025	1	12264.4	7023.0	9990.5	5643.3	17829.5	2150.1	17231.3	8418.1	20872.6	8802.2
		40	60152.2	38021.3	36766.7	47273.8	44378.3	3968.4	47085.1	15657.3	58720.6	2654.3
		100	65961.7	33573.6	37139.4	41497.6	59235.0	2671.5	61871.8	15297.6	33518.0	2224.4
AVERAGE			16428.0	9440.5	10936.4	11058.3	14611.1	1246.9	15778.7	5010.4	13373.9	2028.4

Scores obtained by crossover operators in the case of the auto-adaptation method based on raw scores

N_v	P_{edge}	N_s	CX	LOX	MOX	NWOX	OBX	OX	PBX	PMX	PPX	UPMX
50	0.05	1	0.356	0.329	0.346	0.341	0.411	0.251	0.424	0.331	0.347	0.338
		2	0.434	0.422	0.425	0.394	0.432	0.312	0.433	0.409	0.456	0.359
		5	0.395	0.330	0.336	0.343	0.356	0.280	0.357	0.318	0.331	0.278
100	0.02	1	0.380	0.325	0.366	0.353	0.396	0.190	0.386	0.292	0.390	0.323
		4	0.797	0.718	0.640	0.720	0.696	0.348	0.697	0.644	0.648	0.444
		10	0.865	0.802	0.730	0.787	0.743	0.342	0.756	0.706	0.753	0.461
500	0.0055	1	0.795	0.713	0.839	0.713	1.076	0.305	1.068	0.648	1.015	1.016
		20	2.601	2.532	2.575	2.540	2.670	0.612	2.684	1.983	2.495	0.875
		50	2.563	2.611	2.644	2.606	2.865	0.563	2.874	2.033	2.297	0.885
1000	0.0025	1	0.803	0.671	0.798	0.676	0.990	0.361	0.999	0.574	1.009	0.849
		40	3.149	2.924	2.957	2.929	3.142	0.665	3.146	2.101	3.130	0.693
		100	3.151	3.082	2.770	3.090	3.399	0.558	3.407	2.140	2.299	0.520
AVERAGE			1.357	1.288	1.286	1.291	1.431	0.399	1.436	1.015	1.264	0.587

Success rates of the crossover operators generated by the auto-adaptation method based on success rates

Results

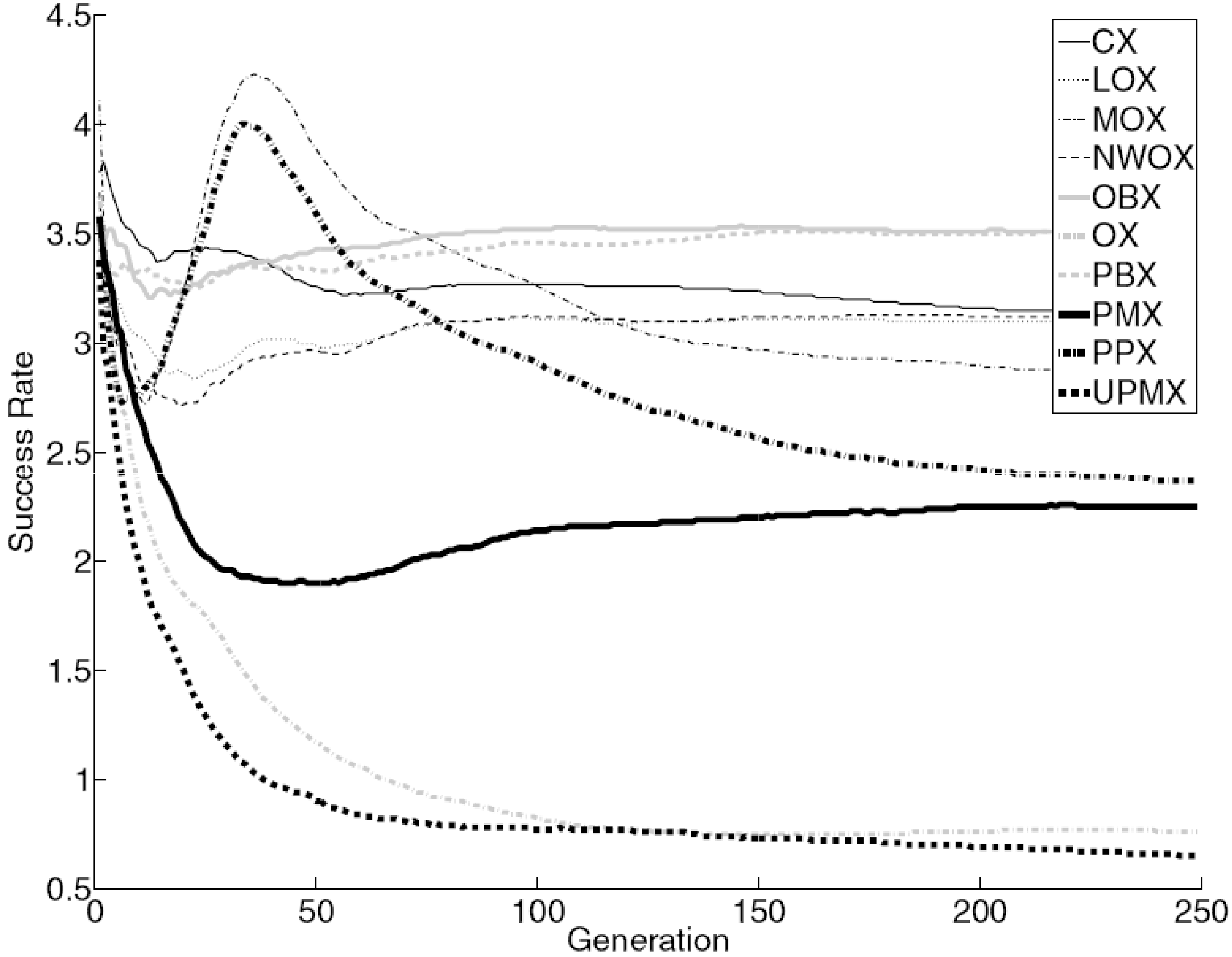
N_v	P_{edge}	N_s	displacement	insertion	inversion	scramble	transpose
50	0.05	1	16.6	8.7	8.1	8.4	2.3
		2	6.3	18.6	3.8	10.8	5.8
		5	10.1	8.0	4.1	3.1	5.0
100	0.02	1	16.3	6.1	22.7	15.3	3.8
		4	17.3	25.3	13.8	26.6	9.9
		10	26.5	23.7	12.3	26.5	10.4
500	0.0055	1	113.7	5.8	111.0	86.8	3.4
		20	121.1	401.7	63.5	62.8	163.9
		50	80.3	581.1	25.8	63.8	124.9
1000	0.0025	1	132.1	6.6	113.8	162.1	3.5
		40	402.8	1107.6	112.5	176.3	353.0
		100	243.2	1165.5	53.0	138.7	339.6
AVERAGE			98.9	279.9	45.4	65.1	85.5

Scores obtained by mutation operators in the case of the auto-adaptation method based on raw scores

N_v	P_{edge}	N_s	displacement	insertion	inversion	scramble	transpose
50	0.05	1	0.054	0.050	0.053	0.033	0.048
		2	0.057	0.086	0.031	0.039	0.058
		5	0.023	0.050	0.021	0.023	0.049
100	0.02	1	0.056	0.036	0.035	0.041	0.022
		4	0.051	0.102	0.058	0.055	0.076
		10	0.068	0.089	0.052	0.067	0.045
500	0.0055	1	0.057	0.019	0.044	0.049	0.010
		20	0.097	0.213	0.060	0.073	0.131
		50	0.074	0.231	0.048	0.051	0.154
1000	0.0025	1	0.049	0.011	0.041	0.040	0.003
		40	0.118	0.275	0.078	0.094	0.186
		100	0.080	0.263	0.043	0.055	0.168
AVERAGE			0.065	0.119	0.047	0.052	0.079

Success rates of the mutation operators generated by the auto-adaptation method based on success rates

Results



The success rates of crossover operators plotted against the generation number for the instance with N_v = 1000 vertices and N_s = 100 fire starting points

Conclusion

- A multiobjective version of the firefighter problem was introduced in this paper
- An evolutionary algorithm with operator autoadaptation was used
- Two autoadaptation stategies were tested:
  - RawScore
  - SuccessRate
- The first mechanism seems to work better on larger instances, while the first one produces better results on smaller instances

Conclusion

- Best crossover operators
  - raw scores → CX crossover
  - success rates → OBX and PBX crossovers
- Best mutation operator: insertion mutation operator
- Success rates of crossover operators vary over the duration of the optimization process
- This motivates using auto-adaptation mechanisms because the performance of any individual operator may deteriorate over certain periods of time

Thank you!